Assignment 1
Systems Neuroscience

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1. **Anatomy**

   Consider a brain with 8 million kilometers of axons and 10^{11} neurons. Assume each neuron contacts 5000 other neurons on average.

   (a) In class you were told that each neuron connects to about 1000 neurons, which seems at odds with 5000 contacts. Explain the discrepancy. (Incompetent instructor is not the answer. At least not in this case.)

   (b) What is the average spacing between contacts on the axons?

   (c) If the average spacing between contacts on dendrites is 2 microns, what is the total lengths of dendrites in the brain?

   (d) If the number of contacts per unit length on axons and dendrites, and the number of contacts per neuron, is about the same for all mammals, how does axon and dendrite length scale with brain size? Assume the density of neurons stays fixed as brain size changes.

2. **Power**

   You have 10^{11} neurons and your brain uses about 20 Watts.

   (a) Assuming the average firing rate is 2 Hz, and that most of the energy is used for spikes, how much energy does a spike cost? (Remember from your physics class that power in Watts $\times$ time in seconds = Joules).

   (b) You can use this number to estimate the resistance, R, of a neuron during a spike. The total energy, E, used during a spike is

$$E = \frac{(\Delta V)^2}{R} \Delta t$$

where ΔV is the difference between rest and peak membrane potential during a spike and Δt is the spike width. Assuming $\Delta t = 1$ ms and $\Delta V = 50$ mV, what is R (in Ohms)?

3. **The linear integrate and fire neuron**

An approximate treatment of spiking neurons is to think of them as passively integrating input and, when the voltage crosses threshold, emitting a spike. This leads to the linear integrate and fire neuron (sometimes called the leaky integrate and fire neuron, and typically abbreviated LIF), which obeys the equation

$$C\frac{dV}{dt} = -g_L(V - \mathcal{E}_L) + I_0(t)\,.$$

This is just the "linear integrate" part. To incorporate spikes, when the voltage gets to threshold (V_t), the neuron emits a spike and the voltage is reset to rest, which for simplicity we'll take to be $\mathcal{E}_L$.

(a) Show that so long as the voltage is below threshold ($V(t) < V_t$), $V(t)$ is given by

$$V(t) = \mathcal{E}_L + [V(0) - \mathcal{E}_L]e^{-t/\tau} + \int_0^t \frac{dt'}{\tau}\, e^{-(t-t')/\tau}\, \frac{I_0(t')}{g_L}$$

where $V(0)$ is the value of the membrane potential at $t = 0$ and τ is the membrane time constant,

$$\tau \equiv \frac{C}{g_L}\,.$$

(b) Show that when $I_0(t) = I_0 =$ constant, $V(t)$ simplifies to

$$V(t) = \mathcal{E}_L + [V(0) - \mathcal{E}_L]e^{-t/\tau} + \frac{I_0}{g_L}\left[1 - e^{-t/\tau}\right].$$

(c) Let $\mathcal{E}_L = -65$ mV, $V_t = -50$ mV, and $V(0) = \mathcal{E}_L$. Sketch $V(t)$ versus t (starting at $t = 0$ and continuing for several time constants, τ) for two cases: $I_0/g_L = -55$ mV and $I_0/g_L = -45$ mV.